Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online): 0493-2137
E-Publication: Online Open Access
Vol:54 Issue:10:2021
DOI: 10.17605/OSF.IO/6F5C3

TOTAL CHROMATIC NUMBER OF EXTENDED DUPLICATE GRAPH OF PATH AND STAR GRAPH FAMILIES

[1]Dr. K. THIRUSANGU,
[1]Department of Mathematics, S.I.V.E.T. College, Gowrivakkam, Chennai-600073, Tamilnadu.
Email id: kthirusangu@gmail.com

M. VIMALA BAI[2], Dr. C. SHOBANA SARMA[3]
[2*, 3]Department of Mathematics, Bharathi Women's College (Autonomous), Chennai-108, Tamilnadu.
Email id: shobanasharma18@gmail.com

Corresponding Author
Email id: mvimalabai24@gmail.com

Abstract

In this paper, we obtain the total chromatic number of middle graph of Extended duplicate graph of path graph $M(EDG[P_n])$, Total graph of Extended duplicate graph of path graph $T(EDG[P_n])$, Middle graph of Extended duplicate graph of star $M(EDG[K_{1,n}])$ and Total graph of extended duplicate graph of a star $T(EDG[K_{1,n}])$.

Keywords: Total coloring, Total chromatic number, Extended duplicate graph, Middle graph, Total graph, Path and Star.

1. Introduction

Throughout this paper, we have considered the finite, simple and undirected graph. Let G = $(V(G), E(G))$ be a graph with the vertex set V(G) and the edge set E(G), respeitively.

A total coloring of G, is a function f: S → C, where S=V(G) ∪E(G) and C is a set of colors to satisfies the given conditions:

(i) no two adjacent vertices receive the same colors,

(ii) no two adjacent edges receive the same colors and

(iii) no edge and its end vertices receive the same colors.

The total chromatic number χ_{tc} of a graph G is the minimum cardinality k such that G may have a total coloring by k colors. The concept of total coloring was introduced by Behzad [1], in 1965.

2. PRELIMINARIES

2.1 Path

A Path in a graph Gis a sequence of vertices such that from each of its vertices there is an edge to the next vertex in the sequence.

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online): 0493-2137
E-Publication: Online Open Access
Vol:54 Issue:10:2021
DOI: 10.17605/OSF.IO/6F5C3

2.2 Star

The star graph is a complete bipartite graph is denoted by$K_{1,n}$.

2.3 Duplicate graph

ADuplicate graph of G ,DG = (V_1, E_1), where the vertex set V_1 = V∪ V' and V ∩ V' =ϕ and f: V → V' is bijective(for v ∈ V, we write f (v) = v' for convenience) and the edge set E_1 of DG is the edge$v_1v_2$is in E if and only if both v_1v_2' and v'_1v_2 are edges in E_1.

2.4 Extended duplicate graph

The extended duplicate graph of DG denoted by EDG, is defined as, add an edge between any two vertex from V to any other vertex in V', except the terminal vertices of V and V'. For convenience, we take v_2' ∈ V and v_2 ∈ $V'$and thus the edge v_2v_2' is joining.

2.5 Middle graph

The middle graph of a graph G denoted by M(G) is define as follows the vertex set of M(G) is V(G) ∪ E(G). Two vertices x, y in M(G) are adjacent if one of the following condition holds:

(i) x, y are in E(G), x and y are adjacent in G.

(ii) x is in V(G), y is in E(G), x and y are incident in G.

2.6 Total graph

The total graph of a graph G, denoted by T(G). The vertex set of T(G) is V(G) ∪ E(G). Two vertices x, y in the vertex set of T(G) are adjacent in T(G) if one of the following condition holds:

(i) x, y are in V(G) and x is adjacent to y in G.

(ii) x, y are in E(G) and x, y are adjacent in G.

(iii) x is in V(G), y is in E(G) and x, y are incident in G.

The extended duplicate graph of path denoted by EDG$[P_n]$ is obtained from the duplicate graph of path by joining v_2 and v_2'.

The extended duplicate graph of star denoted by EDG($K_{1,n}$) is obtained from the duplicate graph of star by joining v_1 and v_1'.

The number of vertices and edges of EDG$[P_n]$is 2n and 2n – 1 respectively.

The number of vertices and edges of EDG$\left[K_{1,n}\right]$ is 2n + 2 and 2n + 1 respectively.

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online): 0493-2137
E-Publication: Online Open Access
Vol:54 Issue:10:2021
DOI: 10.17605/OSF.IO/6F5C3

3. Total Chromatic Number of M(EDG$[P_n]$)

Theorem: 3.1

The total chromatic number of middle graph of extended duplicate graph of path graph is $\chi_{tc}\left(M(EDG[P_n])\right)$= 8for n≥4

Proof

The number of vertices and edges of $M(EDG[P_n])$ is 4n – 1 and 6n – 2 respectively.

The vertex set and the edge set $M(EDG[P_n])$ is given as follows

Case(i): when n is odd

$V\left(M(EDG[P_n])\right) = \{v_i, v_i' : 1 \le i \le n\} \cup \left\{x_i, x_i', u_i, u_i' : 1 \le i \le \left[\frac{n-1}{2}\right]\right\} \cup \{w\}$

$E\left(M(EDG[P_n])\right) = \{v_{2i-1}x_i,\ x_i v_{2i}',\ v_{2i}'u_i,\ u_i v_{2i+1},\ v_{2i-1}'x_i',\ x_i' v_{2i},\ v_{2i}u_i',\ u_i' v_{2i+1}',\ x_i u_i,\ x_i' u_i' :$
$1 \le i \le \left[\frac{n-1}{2}\right]\} \cup \left\{u_i x_{i+1}, u_i' x_{i+1}' : 1 \le i \le \left[\frac{n-3}{2}\right]\right\} \cup \{x_1 w\} \cup \{v_2' w\} \cup$
$\{u_1' w\} \cup \{x_1' w\} \cup \{u_1 w\} \cup \{v_2 w\}$

Case(ii): when n is even

$V\left(M(EDG[P_n])\right) = \{v_i, v_i' : 1 \le i \le n\} \cup \left\{x_i, x_i' : 1 \le i \le \frac{n}{2}\right\} \cup \left\{u_i, u_i' : 1 \le i \le \frac{n}{2} - 1\right\} \cup \{w\}$

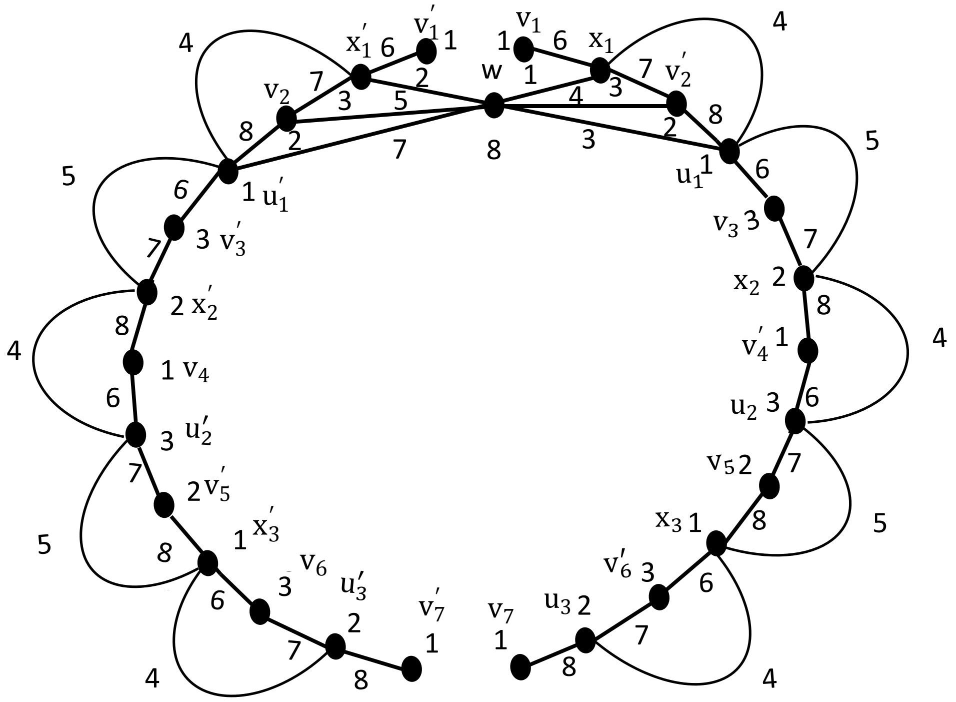
$E\left(M(EDG[P_n])\right) = \left\{v_{2i-1}x_i, x_i v_{2i}', v_{2i-1}'x_i', x_i' v_{2i} : 1 \le\ i \le \frac{n}{2}\right\} \cup \{v_{2i}'u_i, u_i v_{2i+1}, v_{2i}u_i', u_i' v_{2i+1}',$
$x_i u_i, u_i x_{i+1}, x_i' u_i', u_i' x_{i+1}' : 1 \le i \le \left[\frac{n-2}{2}\right]\} \cup \{x_1 w\} \cup \{v_2' w\} \cup \{u_1' w\} \cup$
$\{x_1' w\} \cup \{u_1 w\} \cup \{v_2 w\}$

Now we define total coloring f such that f: S → C.

S = $V\left(M(EDG[P_n])\right) \cup E\left(M(EDG[P_n])\right)$ and C = $\{1, 2, 3, 4, 5, 6, 7, 8\}$

First assign the total coloring for the vertices as follows:

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online): 0493-2137
E-Publication: Online Open Access
Vol:54 Issue:10:2021
DOI: 10.17605/OSF.IO/6F5C3



$\chi_{tc}\big(M(EDG[P_7])\big) = 8$ for n ≥4

$f(w) = 8$

$$f(v_i) = f(v_i') = \begin{cases} 1 & \text{if } i \equiv 1(\text{mod}3) \\ 2 & \text{if } i \equiv 2(\text{mod}3) \\ 3 & \text{if } i \equiv 0(\text{mod}3) \end{cases} \qquad 1 \le i \le n$$

$$f(x_i) = f(x_i') = \begin{cases} 3 & \text{if } i \equiv 1(\text{mod}3) \\ 2 & \text{if } i \equiv 2(\text{mod}3) \\ 1 & \text{if } i \equiv 0(\text{mod}3) \end{cases} \quad \begin{matrix} 1 \le i \le \frac{n}{2} \text{ when n is even} \\ 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil \text{ when n is odd} \end{matrix}$$

$$f(u_i) = f(u_i') = \begin{cases} 1 & \text{if } i \equiv 1(\text{mod}3) \\ 3 & \text{if } i \equiv 2(\text{mod}3) \\ 2 & \text{if } i \equiv 0(\text{mod}3) \end{cases} \quad \begin{matrix} 1 \le i \le \frac{n}{2} - 1 \text{ when n is even} \\ 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil \text{ when n is odd} \end{matrix}$$

Assign the total coloring for the edges as follows

$$f\,(v_{2i-1}x_i) = f\,(v_{2i-1}'x_i') = \begin{cases} 6 & \text{if } i \equiv 1(\text{mod}3) \\ 7 & \text{if } i \equiv 2(\text{mod}3) \\ 8 & \text{if } i \equiv 0(\text{mod}3) \end{cases} \quad \begin{matrix} 1 \le i \le \frac{n}{2} \text{ when n is even} \\ 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil \text{ when n is odd} \end{matrix}$$

$$f\,(u_i v_{2i+1}) = f\,(u_i' v_{2i+1}') = \begin{cases} 6 & \text{if } i \equiv 1(\text{mod}3) \\ 7 & \text{if } i \equiv 2(\text{mod}3) \\ 8 & \text{if } i \equiv 0(\text{mod}3) \end{cases} \quad \begin{matrix} 1 \le i \le \left\lceil \frac{n-2}{2} \right\rceil \text{ when n is even} \\ 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil \text{ when n is odd} \end{matrix}$$

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online): 0493-2137
E-Publication: Online Open Access
Vol:54 Issue:10:2021
DOI: 10.17605/OSF.IO/6F5C3

$$f(x_iv'_{2i}) = f(x'_iv_{2i}) = \begin{cases} 7 & \text{if } i \equiv 1(\text{mod}3) \\ 8 & \text{if } i \equiv 2(\text{mod}3) \\ 6 & \text{if } i \equiv 0(\text{mod}3) \end{cases} \quad \begin{matrix} 1 \le i \le \frac{n}{2} \text{ when n is even} \\ 1 \le i \le \left[\frac{n-1}{2}\right] \text{ when n is odd} \end{matrix}$$

$$f(v'_{2i}u_i) = f(v_{2i}u'_i) = \begin{cases} 8 & \text{if } i \equiv 1(\text{mod}3) \\ 6 & \text{if } i \equiv 2(\text{mod}3) \\ 7 & \text{if } i \equiv 0(\text{mod}3) \end{cases} \quad \begin{matrix} 1 \le i \le \left[\frac{n-1}{2}\right] \text{ when n is even} \\ 1 \le i \le \left[\frac{n-2}{2}\right] \text{ when n is odd} \end{matrix}$$

$$f(x_iu_i) = f(x'_iu'_i) = \begin{cases} 4 & \begin{matrix} 1 \le i \le \left[\frac{n-1}{2}\right] \text{ when n is odd} \\ 1 \le i \le \left[\frac{n-2}{2}\right] \text{ when n is even} \end{matrix} \end{cases}$$

$$f(u_ix_{i+1}) = f(u'_ix'_{i+1}) = \begin{cases} 5 & \begin{matrix} 1 \le i \le \left[\frac{n-3}{2}\right] \text{ when n is odd} \\ 1 \le i \le \left[\frac{n-2}{2}\right] \text{ when n is even} \end{matrix} \end{cases}$$

$f(x_1w) = 1$, $f(x'_1w) = 2$, $f(u_1w) = 3$, $f(u'_1w) = 7$, $f(v_2w) = 5$, $f(v'_2w) = 4$.

It is clear from the above rule of coloring, the graph $M(EDG[P_n])$ is properly total colored with 8 colors.

Hence the total chromatic number of the middle graph of Extended duplicate graph of a path graph $\left(M(EDG[P_n])\right)$ is 8. Therefore $\chi_{tc}\left(M(EDG[P_n])\right) = 8$.

4. Total chromatic number of $T(EDG[P_n])$

Theorem: 4.1

The total chromatic number of Extended duplicate graph of path graph

is given by $\chi_{tc}\left(T(EDG[P_n])\right) = 8$ for $n \geq 4$.

Proof

The number of vertices and edges of $T(EDG[P_n])$ is 4n−1 and 8n−3 respectively.

The vertex set and the edge set of $T(EDG[P_n])$ as follows.

Case(i): If n is odd

$V\left(T(EDG[P_n])\right) = \{v_i, v'_i : 1 \le i \le n\} \cup \left\{x_i, x'_i, u_i, u'_i : 1 \le i \le \left[\frac{n-1}{2}\right]\right\} \cup \{w\}$

$E\left(T(EDG[P_n])\right) = \{v_{2i-1}x_i, x_iv'_{2i}, v'_{2i}u_i, u_iv_{2i+1}, v'_{2i-1}x'_i, x'_iv_{2i}, v_{2i}u'_i, u'_iv'_{2i+1}, x_iu_i, x'_iu'_i,$

$v_{2i-1}v'_{2i}, v'_{2i}v_{2i+1}, v'_{2i-1}v_{2i}, v_{2i}v'_{2i+1} : 1 \le i \le \left[\frac{n-1}{2}\right]\} \cup$

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online): 0493-2137
E-Publication: Online Open Access
Vol:54 Issue:10:2021
DOI: 10.17605/OSF.IO/6F5C3

$$\{u_i x_{i+1}, u_i' x_{i+1}' : 1 \le i \le \left\lceil \frac{n-3}{2} \right\rceil\} \cup \{v_2 v_2'\} \cup \{x_1 w\} \cup \{x_1' w\} \cup \{v_2' w\}$$
$$\cup \quad \{v_2 w\} \cup \{u_1 w\} \cup \{u_1' w\}$$

Case(ii): If n is even

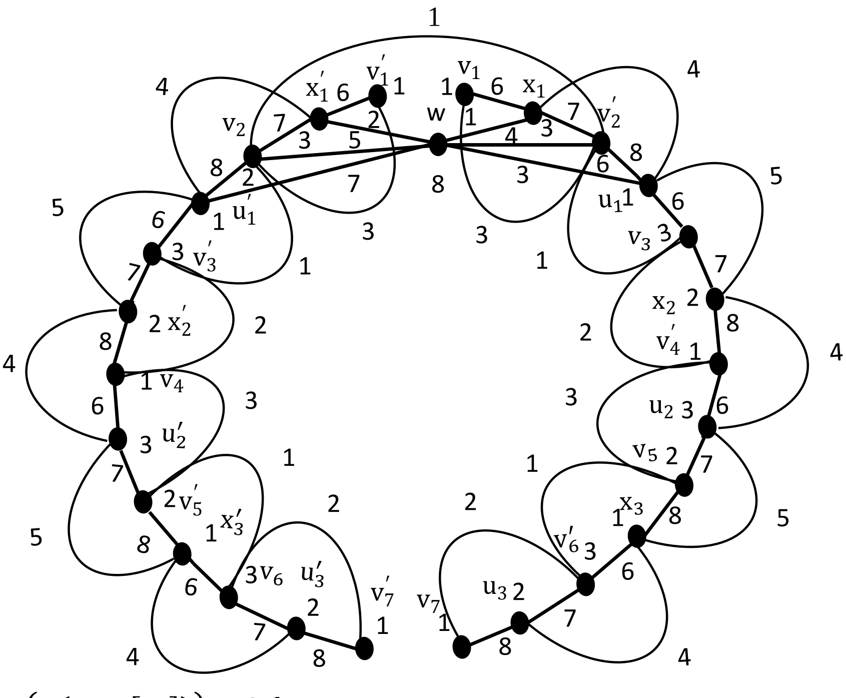
$V(T(EDG[P_n])) = \{v_i, v_i' : 1 \le i \le n\} \cup \left\{x_i, x_i' : 1 \le i \le \frac{n}{2}\right\} \cup \left\{u_i, u_i' : 1 \le i \le \frac{n}{2} - 1\right\} \cup \{w\}$

$E(T(EDG[P_n])) = \left\{v_{2i-1}x_i, x_i v_{2i}', v_{2i-1}v_{2i}', v_{2i-1}' v_{2i}, v_{2i-1}' x_i', x_i' v_{2i} : 1 \le i \le \frac{n}{2}\right\} \cup$

$$\{v_{2i}' u_i, u_i v_{2i+1}, v_{2i} u_i', u_i' v_{2i+1}', x_i u_i, u_i x_{i+1}, x_i' u_i', u_i' x_{i+1}', v_{2i}' v_{2i+1},$$
$$v_{2i} v_{2i+1}' : 1 \le i \le \left\lceil \frac{n-2}{2} \right\rceil\} \cup \{x_1 w\} \cup \{v_2' w\} \cup \{u_1' w\} \cup \{x_1' w\} \cup \{u_1 w\}$$
$$\cup \quad \{v_2 w\} \cup \{v_2 v_2'\}$$

Now we define total coloring f such that f: S → C.

where S = $V(T(EDG[P_n])) \cup E(T(EDG[P_n]))$ and C = $\{1, 2, 3, 4, 5, 6, 7, 8\}$



$\chi_{tc}(T(EDG[P_6]))$ = 8 for n ≥4

First assign the total coloring for the vertices as follows:

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online): 0493-2137
E-Publication: Online Open Access
Vol:54 Issue:10:2021
DOI: 10.17605/OSF.IO/6F5C3

$$f(v_i) = \begin{cases} 1 & \text{if} \quad i \equiv 1 (\text{mod} 3) \\ 2 & \text{if} \quad i \equiv 2 (\text{mod} 3) \\ 3 & \text{if} \quad i \equiv 0 (\text{mod} 3) \end{cases} \qquad 1 \le i \le n$$

$$f(v'_1) = 1, \; f(v'_2) = 6, \; f(w) = 8$$

$$f(v'_i) = \begin{cases} 3 & \text{if} \quad i \equiv 0 (\text{mod} 3) \\ 1 & \text{if} \quad i \equiv 1 (\text{mod} 3) \\ 2 & \text{if} \quad i \equiv 2 (\text{mod} 3) \end{cases} \qquad 3 \le i \le n$$

$$f(x_i) = f(x'_i) = \begin{cases} 3 & \text{if} \quad i \equiv 1 (\text{mod} 3) \\ 2 & \text{if} \quad i \equiv 2 (\text{mod} 3) \\ 1 & \text{if} \quad i \equiv 0 (\text{mod} 3) \end{cases} \qquad \begin{array}{l} 1 \le i \le \left[\frac{n-1}{2}\right] \text{when n is odd} \\ 1 \le i \le \frac{n}{2} \text{when n is even} \end{array}$$

$$f(u_i) = f(u'_i) = \begin{cases} 1 & \text{if} \quad i \equiv 1 (\text{mod} 3) \\ 3 & \text{if} \quad i \equiv 2 (\text{mod} 3) \\ 2 & \text{if} \quad i \equiv 0 (\text{mod} 3) \end{cases} \qquad \begin{array}{l} 1 \le i \le \left[\frac{n-1}{2}\right] \text{ when n is odd} \\ 1 \le i \le \frac{n}{2} - 1 \text{ when n is even} \end{array}$$

Now assign the total coloring for the edges as follows:

$$f(v_{2i-1}x_i) = f(v'_{2i-1}x'_i) = \begin{cases} 6 & \text{if} \quad i \equiv 1 (\text{mod} 3) \\ 7 & \text{if} \quad i \equiv 2 (\text{mod} 3) \\ 8 & \text{if} \quad i \equiv 0 (\text{mod} 3) \end{cases} \qquad \begin{array}{l} 1 \le i \le \frac{n}{2} \text{ when n is even} \\ 1 \le i \le \left[\frac{n-1}{2}\right] \text{when n is odd} \end{array}$$

$$f(u_i v_{2i+1}) = f(u'_i v'_{2i+1}) = \begin{cases} 6 & \text{if} \quad i \equiv 1 (\text{mod} 3) \\ 7 & \text{if} \quad i \equiv 2 (\text{mod} 3) \\ 8 & \text{if} \quad i \equiv 0 (\text{mod} 3) \end{cases} \qquad \begin{array}{l} 1 \le i \le \left[\frac{n-2}{2}\right] \text{when n is even} \\ 1 \le i \le \left[\frac{n-1}{2}\right] \text{when n is odd} \end{array}$$

$$f(x_i v'_{2i}) = f(x'_i v_{2i}) = \begin{cases} 7 & \text{if} \quad i \equiv 1 (\text{mod} 3) \\ 8 & \text{if} \quad i \equiv 2 (\text{mod} 3) \\ 6 & \text{if} \quad i \equiv 0 (\text{mod} 3) \end{cases} \qquad \begin{array}{l} 1 \le i \le \frac{n}{2} \text{when n is even} \\ 1 \le i \le \left[\frac{n-1}{2}\right] \text{when n is odd} \end{array}$$

$$f(v'_{2i}u_i) = f(v_{2i}u'_i) = \begin{cases} 8 & \text{if} \quad i \equiv 1 (\text{mod} 3) \\ 6 & \text{if} \quad i \equiv 2 (\text{mod} 3) \\ 7 & \text{if} \quad i \equiv 0 (\text{mod} 3) \end{cases} \qquad \begin{array}{l} 1 \le i \le \left[\frac{n-1}{2}\right] \text{when n is odd} \\ 1 \le i \le \left[\frac{n-2}{2}\right] \text{when n is even} \end{array}$$

$$f(x_i u_i) = f(x'_i u'_i) = \begin{cases} 4 & \begin{array}{l} 1 \le i \le \left[\frac{n-1}{2}\right] \text{when n is odd} \\ 1 \le i \le \left[\frac{n-2}{2}\right] \text{when n is even} \end{array} \end{cases}$$

$$f(u_i x_{i+1}) = f(u'_i x'_{i+1}) = \begin{cases} 5 & \begin{array}{l} 1 \le i \le \left[\frac{n-3}{2}\right] \text{when n is odd} \\ 1 \le i \le \left[\frac{n-2}{2}\right] \text{when n is even} \end{array} \end{cases}$$

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online): 0493-2137
E-Publication: Online Open Access
Vol:54 Issue:10:2021
DOI: 10.17605/OSF.IO/6F5C3

$$f(v_{2i-1}v'_{2i}) = f(v'_{2i-1}v_{2i}) = \begin{cases} 3 & \text{if} \quad i \equiv 1(\text{mod}3) \\ 2 & \text{if} \quad i \equiv 2(\text{mod}3) \\ 1 & \text{if} \quad i \equiv 0(\text{mod}3) \end{cases} \quad \begin{matrix} 1 \leq i \leq \left\lceil\frac{n-1}{2}\right\rceil \text{ when n is odd} \\ 1 \leq i \leq \left\lceil\frac{n-2}{2}\right\rceil \text{ when n is even} \end{matrix}$$

$$f(v'_{2i}v_{2i+1}) = f(v_{2i}v'_{2i+1}) = \begin{cases} 1 & \text{if} \quad i \equiv 1(\text{mod}3) \\ 3 & \text{if} \quad i \equiv 2(\text{mod}3) \\ 2 & \text{if} \quad i \equiv 0(\text{mod}3) \end{cases} \quad \begin{matrix} 1 \leq i \leq \left\lceil\frac{n-1}{2}\right\rceil \text{ when n is odd} \\ 1 \leq i \leq \left\lceil\frac{n-2}{2}\right\rceil \text{ when n is even} \end{matrix}$$

$f(x_1w) = 1$, $f(x'_1w) = 2$, $f(v_2w) = 5$, $f(v'_2w) = 4$, $f(u'_1w) = 7$, $f(u_1w) = 3$, $f(v_2v'_2) = 1$.

It is clear from the above rule of coloring, the graph $T(EDG[P_n])$ is properly total colored with 8 colors.

Hence the total chromatic number of the total graph of Extended duplicate graph of path graph $\left(T(EDG[P_n])\right)$ is 8.

Therefore $\chi_{tc}\left(M(EDG[P_n])\right) = 8$.

5. Total Chromatic Number of $\mathbf{M(EDG[K_{1,n}])}$.

Theorem 5.1

The total chromatic number of middle graph of extended duplicate graph of star graph is given by

$\chi_{tc}\left(M\left(EDG[K_{1,n}]\right)\right) = n+4$ for $n \geq 3$

Proof:

The number of vertices and edges of middle graph of extended duplicate graph of star graph $M\left(EDG[K_{1,n}]\right)$ 4n+3 and $n^2 + 3n + 4$ respectively.

$V\left(M\left(EDG[K_{1,n}]\right)\right) = \{V, x'_1, x'_2 \dots x'_n\} \cup \{V', x_1, x_2 \dots x_n\} \cup \{v'_1, v'_2 \dots v'_n\} \cup \{v_1, v_2, \dots v_n\} \cup \{w\}$

$E\left(M\left(EDG[K_{1,n}]\right)\right) = \{Vx'_i, V'x_i : 1 \leq i \leq n\} \cup \{x_iv_i, x'_iv'_i : 1 \leq i \leq n\} \cup \{v'_1w\} \cup \{v_1w\} \cup \{x'_1w\} \cup \{x_1w\} \cup \{x_ix_j : 1 \leq i < j \leq n\}$

In $M\left(EDG[K_{1,n}]\right)$ the vertices $\{V, x'_1, x'_2 \dots x'_n\}$ and $\{V', x_1, x_2 \dots x_n\}$ induce a clique of order n+1.

Now we define the total coloring f such that f: S → C as follows.

$S = V\left(M\left(EDG[K_{1,n}]\right)\right) \cup E\left(M\left(EDG[K_{1,n}]\right)\right)$ and C = {1, 2, 3 … n+4}

First assign the total coloring for the vertices as follows:

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online): 0493-2137
E-Publication: Online Open Access
Vol:54 Issue:10:2021
DOI: 10.17605/OSF.IO/6F5C3

f (V) = f (V′) = f (w) = n + 4

$f(v_i)$ = n+1 : $1 \leq i \leq n$

f (v_i') = n+2 : $1 \leq i \leq n$

f (x_i) = f (x_i') = i for $1 \leq i \leq n$

Now assign the total coloring for the edges as follows:

f $(x_i v_i)$ = f $(x_i' v_i')$ = i–1 $2 \leq i \leq n$

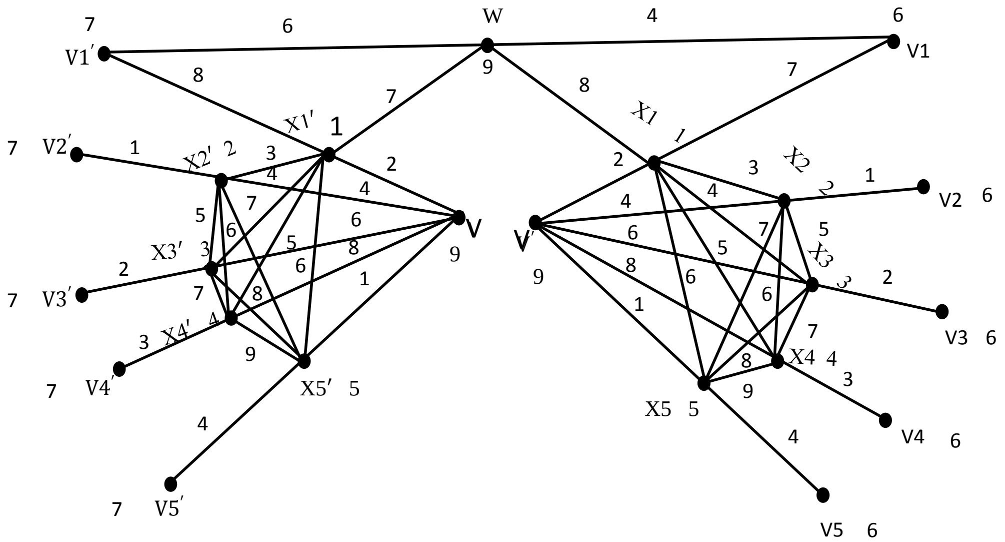
f (Vx_i') = f $(V'x_i)$ = $\begin{cases} 2i\,(\mathrm{mod}(n+2)) & \text{if } 2i \not\equiv 0 \bmod (n+2) \\ n+2 & \text{otherwise} \end{cases}$ when n is odd

f (Vx_i') = f $(V'x_i)$ = $\begin{cases} 2i\,(\mathrm{mod}(n+3)) & \text{if } 2i \not\equiv 0 \bmod (n+3) \\ n+3 & \text{otherwise} \end{cases}$ when n is even

f $(x_i x_j)$ = f $(x_i' x_j')$ = $\begin{cases} i+j\,(\mathrm{mod}(n+2)) & \text{if } (i+j) \not\equiv 0 \bmod (n+2) \\ n+2 & \text{otherwise} \end{cases}$ when n is odd

f $(x_i x_j)$ = f $(x_i' x_j')$ = $\begin{cases} i+j \bmod (n+3) & \text{if } (i+j) \not\equiv 0 \bmod (n+3) \\ n+3 & \text{otherwise} \end{cases}$ when n is even

f $(x_1' v_1')$ = n+3, f $(v_1' w)$ = n+1, f $(x_1' w)$ = n+2, f $(w v_1)$ = n – 1, f $(w x_1)$ = n+3, f $(x_1 v_1)$ = n+2.



$\chi_{tc}\left(M\left(EDG\left[K_{1,5}\right]\right)\right)$ = 9 for n≥3

It is clear from the above rule of coloring the graph $M(EDG[K_{1,n}])$ is properly total colored with n+4 colors.

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online): 0493-2137
E-Publication: Online Open Access
Vol:54 Issue:10:2021
DOI: 10.17605/OSF.IO/6F5C3

Hence the total chromatic number of middle graph of extended duplicate graph of a star graph$\left(M\left(EDG\left[K_{1,n}\right]\right)\right)$ is n + 4.

Therefore $\chi_{tc}\left(M\left(EDG\left[K_{1,n}\right]\right)\right)$ = n+4 for n≥3.

6. Total Chromatic number of T$\left(EDG\left[K_{1,n}\right]\right)$

Theorem 6.1

The total chromatic number of total graph of extended duplicate graph of star graph is given by
$\chi_{tc}\left(T\left(EDG\left[K_{1,n}\right]\right)\right)$ = 2n+1 for n≥3.

Proof

The number of vertices and edges of total graph of extended duplicate graph of $K_{1,n}$ is4n + 3 and$n^2 + 5n + 5$ respectively.

$V\left(T\left(EDG\left[K_{1,n}\right]\right)\right) = \{V, x'_1, x'_2 \dots x'_n\} \cup \{V', x_1, x_2 \dots x_n\} \cup \{v'_1, v'_2 \dots v'_n\} \cup \{v_1, v_2, \dots v_n\} \cup \{w\}$

$E\left(T\left(EDG\left[K_{1,n}\right]\right)\right) = \{Vx'_i, V'x_i : 1 \le i \le n\} \cup \{v'_1 w\} \cup \{v_1 w\} \cup \{v_1 v'_1\} \cup \{x'_1 w\} \cup \{x_1 w\} \cup$

$\{x_i x_j : 1 \le i < j \le n\}$

In $T\left(EDG\left[K_{1,n}\right]\right)$ the vertices $\{V, x'_1, x'_2 \dots x'_n\}$ and $\{V', x_1, x_2 \dots x_n\}$ induce a clique of order n+1.

Now we define the total coloring f such that f: S → C as follows.

$S = V\left(T\left(EDG\left[K_{1,n}\right]\right)\right) \cup E\left(T\left(EDG\left[K_{1,n}\right]\right)\right)$ and C = {1, 2, 3 … 2n+1}

First assign the total coloring for the vertices as follows:

f (V) = f (V′) = f (w) = 2n+1

$f(x_i) = f(x'_i) = i$ for $1 \le i \le n$

$f(v_i) = 2n$, $f(v'_1) = 2n-1$

$f(v'_i) = 2n$: $2 \le i \le n$

Now assign the total coloring for the edges as follows:

$f(x_i v_i) = f(x'_i v'_i) = i-1$ $\quad 2 \le i \le n$

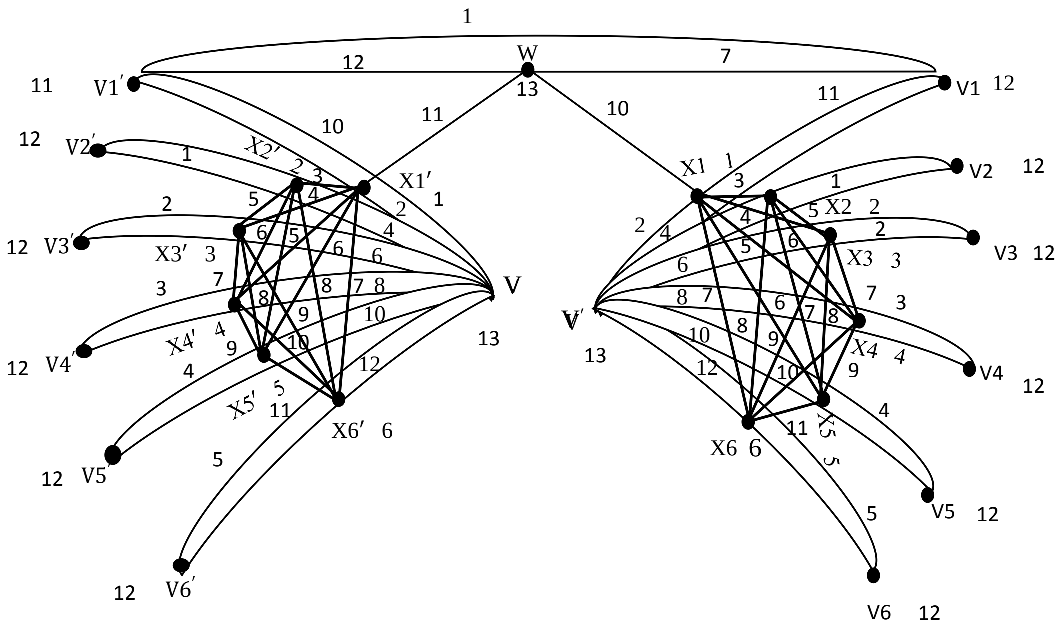
$$f(Vx'_i) = f(V'x_i) = \begin{cases} 2i\ (\mathrm{mod}(2n)) & \text{if } 2i \not\equiv 0 \bmod(2n) \\ 2n & \text{otherwise} \end{cases}$$

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online): 0493-2137
E-Publication: Online Open Access
Vol:54 Issue:10:2021
DOI: 10.17605/OSF.IO/6F5C3

$$f(Vv_i') = f(V'v_i) = \begin{cases} 2i-1\ (\mathrm{mod}(2n-1)) & \text{if } 2i-1 \not\equiv 0 \ \mathrm{mod}(2n-1) \\ 2n-1 & \text{otherwise} \end{cases}$$

$$f(x_i x_j) = \begin{cases} (i+j)\ (\mathrm{mod}(2n+1)) & \text{if } i+j \not\equiv 0 \ \mathrm{mod}(2n+1) \\ 2n+1 & \text{otherwise} \end{cases}$$

$f(x_1'v_1') = 2n-2$, $f(x_1'w) = 2n-1$, $f(v_1'w) = n$, $f(wv_1) = n+1$, $f(wx_1) = 2n-2$, $f(x_1v_1) = 2n-1$, $f(v_1v_1') = 1$.



$\chi_{tc}\left(T(EDG[K_{1,n}])\right)$ = 2n+1 for n≥3.

It is clear from the above rule of coloring the graph $T(EDG[K_{1,n}])$ is properly total colored with 2n + 1colors.

Hence the total chromatic number of total graph of Extended duplicate graph of a star graph $T(EDG[K_{1,n}])$ is 2n+1. Therefore $\chi_{tc}\left(T(EDG[K_{1,n}])\right)$ = 2n+ 1 for n≥4.

7. CONCLUSION

We have obtained determined the total chromatic number for the following graphs.

(i)	$\chi_{tc}(M(EDG[P_n]))$ = 8	n≥4
(ii)	$\chi_{tc}(T(EDG[P_n]))$ = 8	n≥4
(iii)	$\chi_{tc}\left(M(EDG[K_{1,n}])\right)$ = n + 4	for n≥ 3
(iv)	$\chi_{tc}\left(T(EDG[K_{1,n}])\right)$ = 2n + 1	for n≥ 3

Tianjin Daxue Xuebao (Ziran Kexue yu Gongcheng Jishu Ban)/
Journal of Tianjin University Science and Technology
ISSN (Online): 0493-2137
E-Publication: Online Open Access
Vol:54 Issue:10:2021
DOI: 10.17605/OSF.IO/6F5C3

REFERENCES

[1] Behzad M., Graphs and their chromatic numbers, Doctoral Thesis, Michigan state university (1965).

[2] Behzad M., Total concepts in graph theory, Arscombin, 23, (1987), 35 - 40.

[3] C. ShobanaSarma , and K. Thirusangu, Acyclic coloring of Extended Duplicate Graph of Star Graph Families, International Journal of Mathematics Trends and Technology, (2018) 109 - 112.

[4] Behzad M., A criterion for the planarity of the total graph of a graph, Proc. Cambridge philos. soc.,63, (1967), 679 - 681.

[5] I. Hamada and I. Yoshimura, Travels ability and connectivity of middle graph of a graph, Dis. Math, 17 (1976) 247 - 255.

[6] D. Muthuramakrishnan and G. Jayaraman, Total Chromatic Number of Middle and Total graph of path and Sunlet Graph, International Journal of Scientific and Innovative Mathematical Research (2018).

[7] D. Muthuramakrishnan, Total Chromatic Number of Star and Bistar Graphs, International Journal of Pure and Applied Mathematics, (2017) 699 - 708.

[8[J.A.Bondy and U.S.R.Murty, “Graph Theory With Applications”,MacMillan,London,976.

[9] K.Thirusangu,P.P.Ulaganathan And B.Selvam, “Cordial Labeling In Duplicate Graphs”,Int.J.Computer Math.Sci.Appl.,4(1-2) (2010) 179-186.